HOLYWELL CHURCH OF ENGLAND MIDDLE SCHOOL

Numeracy and Calculation Policy for Holywell School

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Responsibility	All staff and the governing body
Effective Date	November 2024
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Storage; (i) Electronic (ii) Hard Copy	(i) School network and on school website (ii) Policy file.
Distribution	All staff Parents

Rationale
The following calculation policy has been devised to meet requirements of the National Curriculum for the teaching and learning of mathematics, and is also designed to give students a consistent and smooth progression of learning in calculations across the school.

Aims
The Numeracy and Calculation Policy aims to ensure all students:
- understand important concepts and make connections within mathematics
- show high levels of fluency in performing written and mental calculations
- are taught consistent calculation strategies
- are ready for the next stage of learning
- have a smooth transition between phases
- are able to add, subtract, multiply and divide efficiently
- are competent in fluency, reasoning and problem solving.

Our approach
We feel that it is fundamental for children to be able to move from conceptual learning to abstract learning in order to be able to successfully understand, use and apply their mathematical skills. The calculation strategies which will be used will reflect this ideology – moving from concrete to pictorial and then abstract recording (CPA), leading to more formal written methods. Mental methods and strategies will work in partnership with these methods.

Selecting the methods
We have considered the following factors when selecting the calculation strategies to be used:
- research and evidence
- building on experiences in Lower School
- consistency and progression across the school
- ability to apply mental methods
- an emphasis on understanding the concept rather than relying on the procedure
- written methods that can be applied across all four operations

- adopting the Concrete, Pictorial and Abstract approach (CPA)
- methods that can be followed through to algebraic representation.

Providing a context for calculation:
It is important that any type of calculation is given a real-life context or problem-solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Other methods
We recognise that there are many successful written methods in use today. However, we know from the evidence collected and the research we have been involved in, that the emphasis should remain on understanding the concept (relational learning) and operation rather than procedural (instrumental) learning (Skemp 2012).

Numeracy is…
… much more than just knowing about numbers and number operations. It requires practical understanding and encourages the inclination to problem solve. Numeracy develops and enhances an analytical approach in dealing with measurement and handling data.

Important points to note:
- Mental arithmetic should be recommended as a first resort. Teachers are encouraged to seek and compare a range of calculation methods, by asking students how they worked out a calculation and insisting everyone listens and responds positively to the responses. [link with Literacy across the curriculum.]
- As a result of the primary Numeracy initiative students are far more confident in carrying out calculations mentally and should be encouraged to continue and develop these skills in KS3… and throughout KS4 and beyond.
- Students will gain more and remember much more if understanding is given prominence.
- Students should be helped to develop their own methods of calculation, rather than be taught different set procedures.
- Students are expected to have their own calculator, pair of compasses and protractor.

Using and Applying Mathematics
In 'Using and Applying mathematics' to solve problems, students use a variety of thinking skills which should be transferable to other subject areas. These include:
  - breaking the problem down into more manageable parts
  - logical deduction
  - hypothesising
  - predicting and testing.

Calculators
- Use of calculators allows freedom from repetitive difficult calculations. Students should have open access to calculators (preferably their own) but be encouraged to use them sensibly e.g. not for working out simple calculations. (NB. Calculators can be provided for disadvantaged students; we will also consider requests for providing personal calculators to those who need/request them)
- It is good practice to always estimate answers before using a calculator.
- Sensible rounding is expected. (Staff to advise re subject requirements)
- Students should be encouraged to set down methods working, whether using a calculator or not. Answers only are not acceptable.
- Care must be taken when students are using basic calculators as the order of operations is often not always in-built (BIDMAS). New scientific calculators often do calculations in the order they are entered eg sine 30, 50 …

Number

- In all arithmetic, the importance of place value should be stressed.
- It is better to present sums initially in a horizontal format, to encourage some form of mental calculation or estimation.
- Language involving plus/positive and minus/negative often causes confusion. All of these terms should be used regularly.
- When referring to decimals, say "three point one four" rather than "three point fourteen".
- In a line of working, an "equals" sign should appear only once. Working should develop down the page, with equals signs in line (The following is **poor** practice:
  6 x (3 + 4) = 7 = 6 x 7 = 42, as students are equating unequal things.)
- Emphasise the link between fractions, decimals, ratios and percentages. The % button on a calculator needs to be used with care. Note, however, that the fraction button is very useful.
- The correct written form of numbers in standard form must be used, i.e. a calculator display of 1.5763^{04} must be written as 1.5763×10^4.

Algebra

- Take care when using terms like "cross multiply" and "swap sides - swap signs" as these can lead to misunderstandings. Instead, use the balance method. (See a member of the Mathematics Team for more detail.)
- Running through a formula with " easy" numbers may aid student understanding.
- Trial and improvement is an acceptable mathematical method.

Shape Space and Measures

- The word "similar" in mathematics is used to describe objects that are exactly the same shape, but not necessarily the same size - one object is an exact scaled version of the other.
- Work is done in mathematics on common Imperial units and their metric equivalents. Technology needs students to be particularly familiar with millimetres.
- Appropriate units must always be stated; e.g. in answers, graph axes etc.
- Try not to add to the confusion of mass and weight

  Mass is a measure of the amount of substance and is measured in kg. On planet Earth, 1 kg of anything exerts a force of 10N in the direction of the Earth - its weight. This is due to the gravitational pull of the Earth. On other planets or the Moon, the gravitational pull will be different and so the force exerted by 1 kg will vary, e.g. in outer space there is virtually no gravitational pull, you would be 'weightless'. When you stand on the bathroom scales your weight, i.e. force, compresses a spring. The manufacturers create a display that converts the amount of compression into mass, i.e. the compression due to 10N reads as 1 kg on the display. This is the simplest way of determining mass.

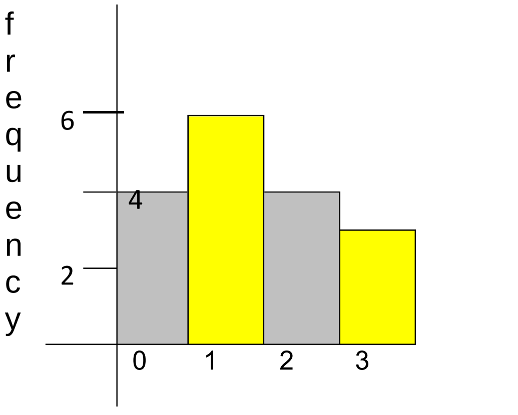
  So use the term mass instead of weight.
- We use the following language for bearings:
  - bearings always start with 0^o from North
  - bearings are always measured clockwise
  - bearings need the ^{o} (degree) symbol
  - bearings need 3 figures.

Handling Data

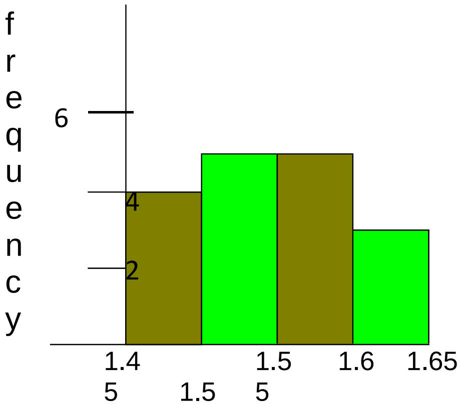
- Always use degrees when constructing pie charts; label sectors with the data or a key.
- All graphs should have a title and labelled axes, with units marked.
- When interpreting graphs, make sure students know what each "small square" represents on **each** axis.

- Encourage students to always consider whether an information graph axis should or should not start from zero in a particular case; and the implication of this.
- Bar charts are used to display discrete data (data which is counted). Histograms have no gaps and are used to display continuous data (data which is measured).

Note the labelling of the axes:



number of brothers and sisters



height (in m)

- When using the term "average" please say "mean average" (or mode or median).
- Probabilities should be written as fractions, decimals or percentages and definitely not as "1 in 7" or "1 out of 7" or "1:7".
- When reading off the gradient of a line, ensure that students have a full understanding of the scale on each axis.
- Line graphs should be straight lines drawn with a ruler and pencil.

When are children ready for written calculations?

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

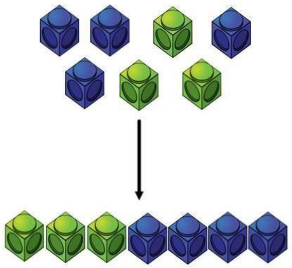
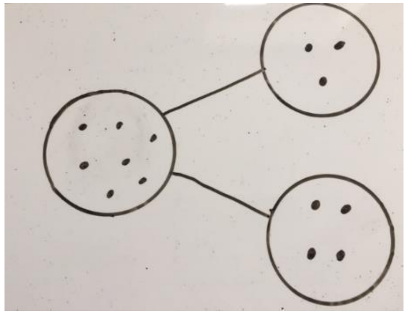
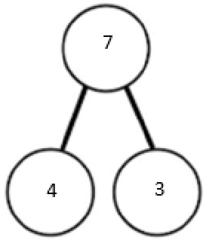
Multiplication and division

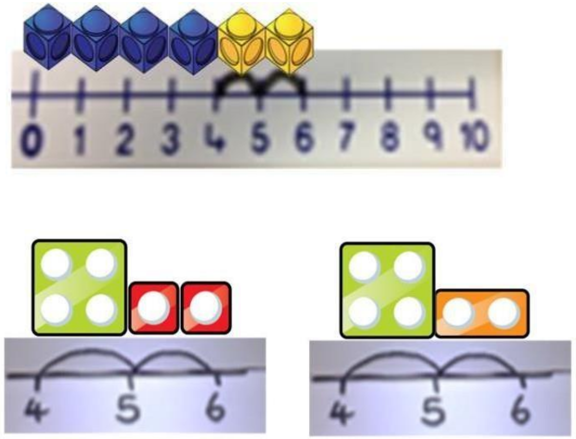
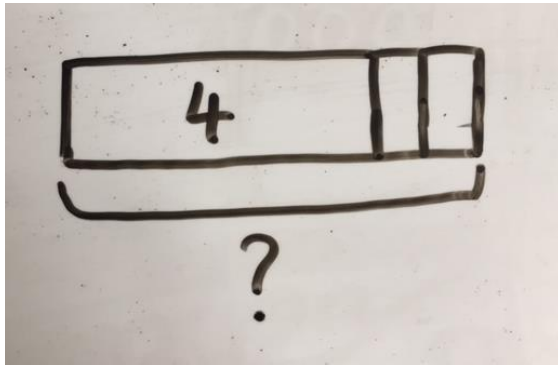
- Do they know the 2, 3, 4, 5- and 10-times table?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two- and three-digit numbers by 10 and 100?
- Can they double and halve two-digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

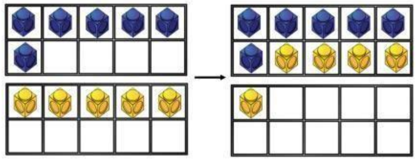
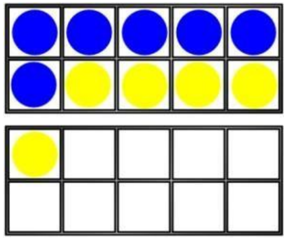
The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

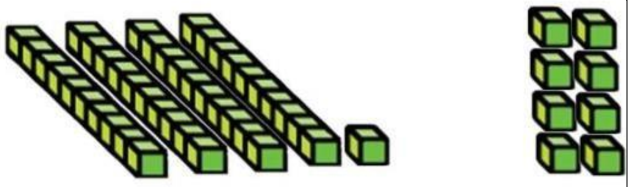
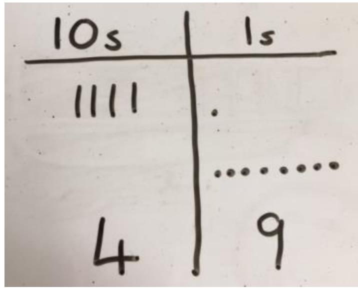
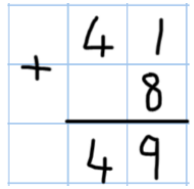
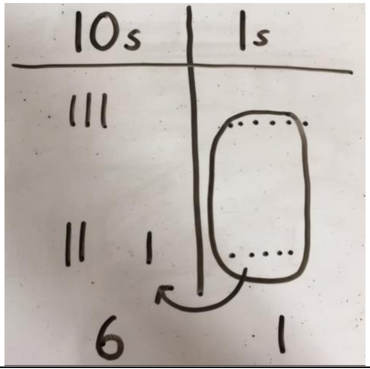
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’ ‘is the same as’.

Combining two parts to make a whole (useother resourcestooe.g.eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Fourisa part,3isapartandthe whole is seven. 

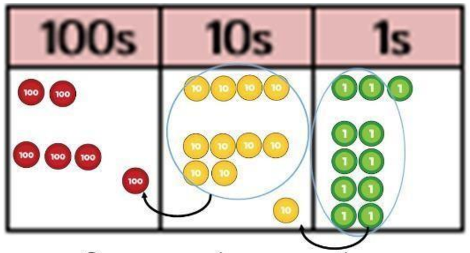
<table>
<tr>
<td>Counting on using number lines using cubes or Numicon.

</td>
<td>A bar model which encourages the children to count on, rather than count all.

</td>
<td>The abstract number line: What is 2 more than 4? What is the sum of 2and4? What is the total of 4 and2? 4 + 2

</td>
</tr>
<tr>
<td>Regrouping to make 10; using ten frames and counters/cubes or using Numicon.
6 + 5
</td>
<td>Children to draw the ten frame and counters/cubes.
</td>
<td>Children to develop an understanding of equality e.g.
$6 + \square = 11$
$6 + 5 = 5 + \square$
$6 + 5 = \square + 4$</td>
</tr>
</table>

<table>
<tr>
<td>TO + O using base 10. Continue to develop understanding of partitioning and place value.
41 + 8
</td>
<td>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.

</td>
<td>41 + 8
1 + 8 = 9
40 + 9 = 49

</td>
</tr>
<tr>
<td>TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25</td>
<td>Children to represent the base 10 in a place value chart.

</td>
<td>Looking for ways to make 10.

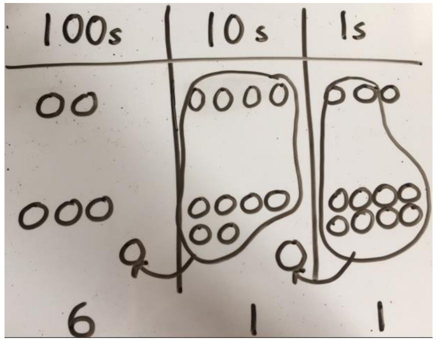
30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61</td>
</tr>
</table>

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



6 1 1

Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline {}_{1\ 1} \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34

?

21 34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

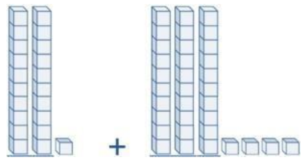
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

= 21 + 34

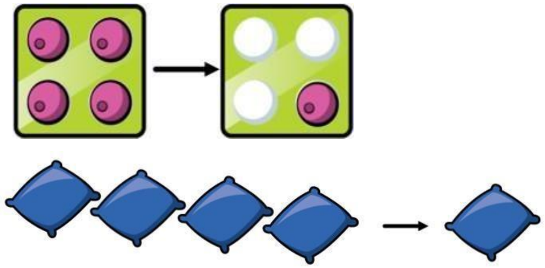
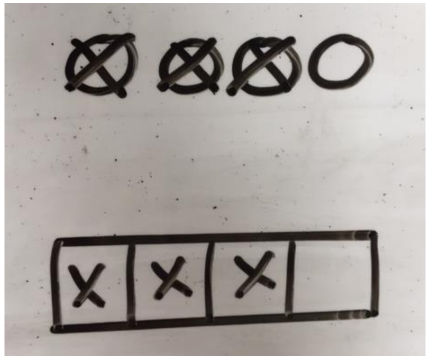
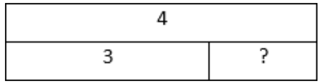
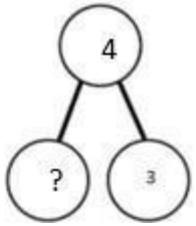
Calculate the sum of twenty- one and thirty-four.



Missing digit problems:

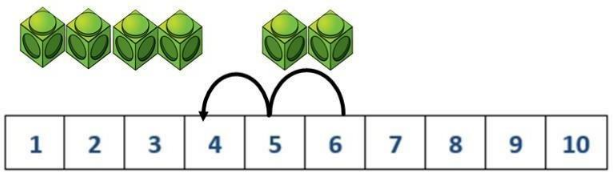
Calculation policy: Subtraction

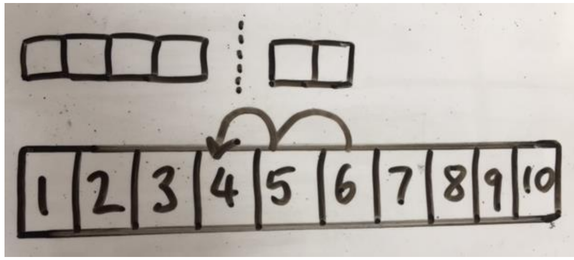
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

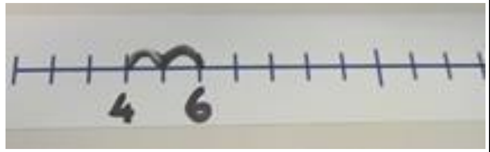
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	**4- 3 =** **= 4 – 3**  

<table>
<tr>
<td>Counting back (using number lines or numbertracks) children start with 6 and count back 2.

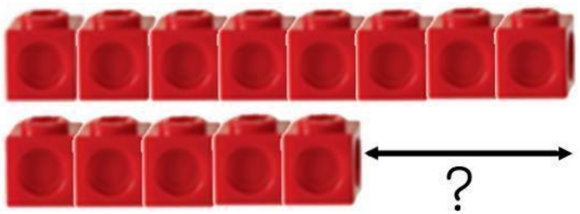
6 – 2 = 4

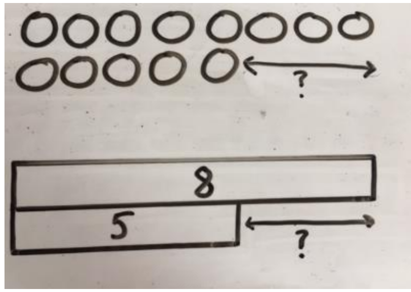

</td>
<td>Children to represent what they see pictorially e.g.


</td>
<td>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line


</td>
</tr>
<tr>
<td>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.

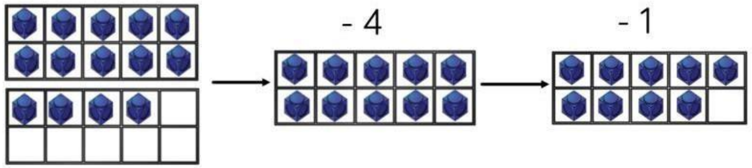
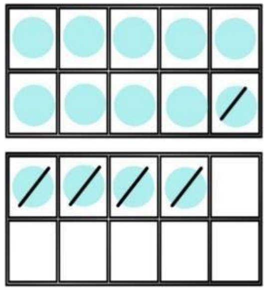
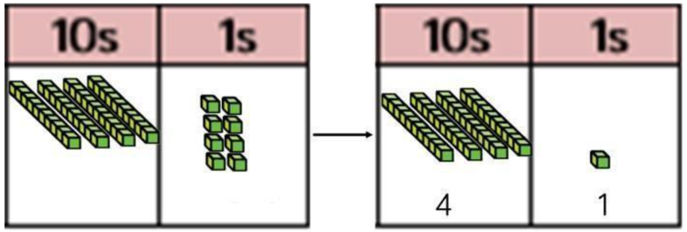
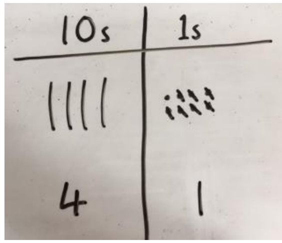
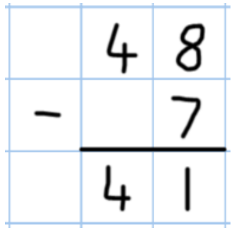

</td>
<td>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.

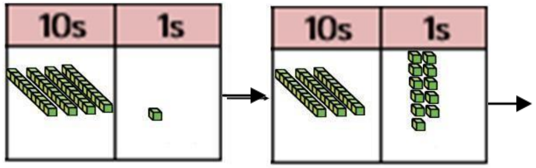
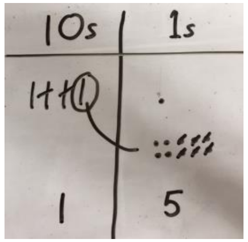
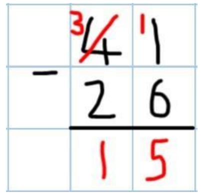
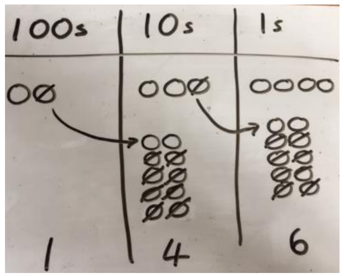
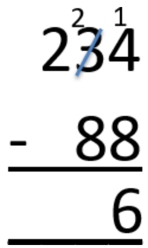

</td>
<td>Find the difference between 8

and 5. 8 – 5, the difference

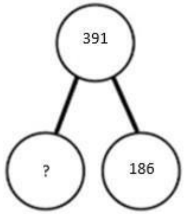
is

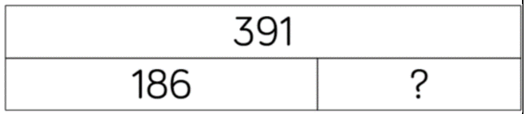
Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.</td>
</tr>
</table>

Making 10 using ten frames. 14 – 5  	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend. 14 - 5 = 9 4 1 14 – 4 =10 10 – 1 = 9
Column method using base 10. 48-7  	Children to represent the base 10 pictorially.  	Column method or children could count back 7. 

Column method using base 10 and having to exchange. 41 – 26  	Represent the base 10 pictorially, remembering to show the exchange.  	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Column method using place value counters. 234 – 88	Represent the place value counters pictorially; remembering to show what has been exchanged.  	Formal column method. Children must understand what has happened when they have crossed out digits. 

Conceptual variation; different ways to ask children to solve 391 - 186





Raj spent £391, Timmy spent £186. How much more did Raj spend?

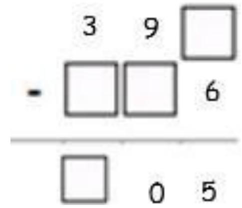
Calculate the difference between 391 and 186.

= 391 – 186

```
 391
-186
____
```

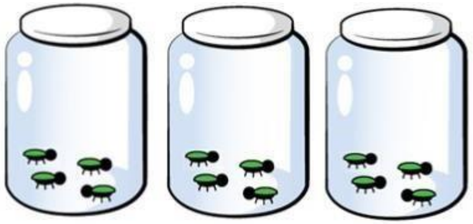
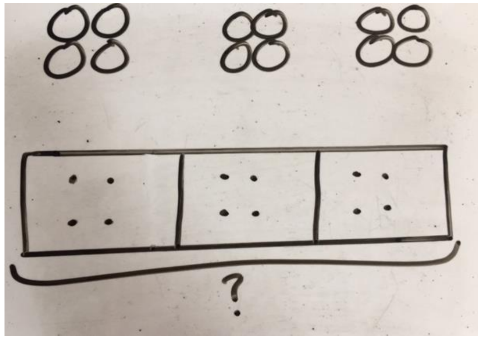
What is 186 less than 391?

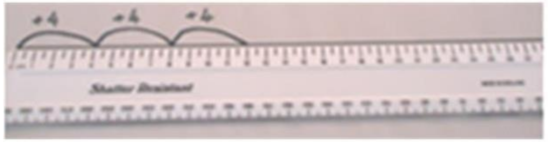
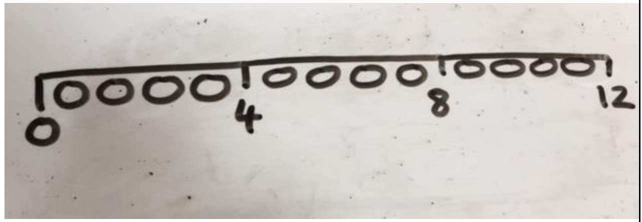
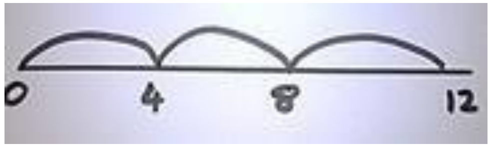
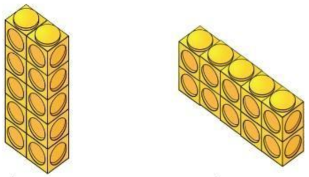
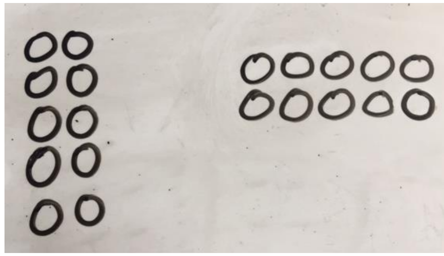
Missing digit calculations

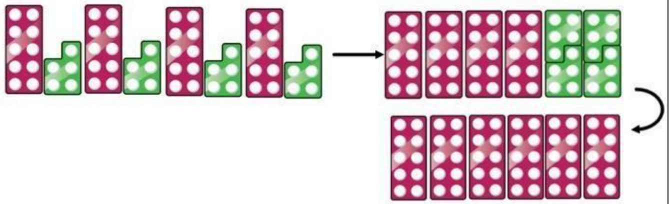
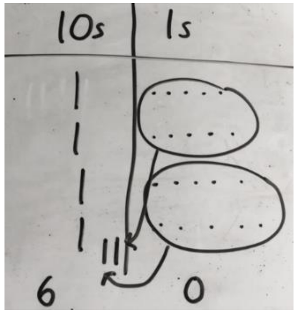
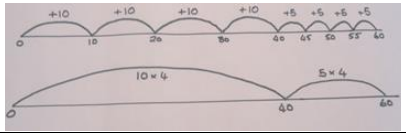
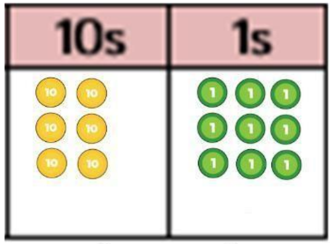
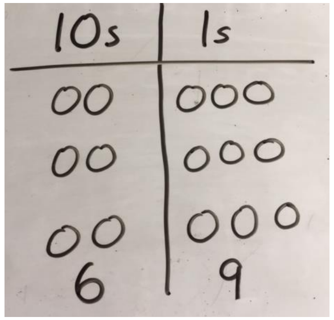


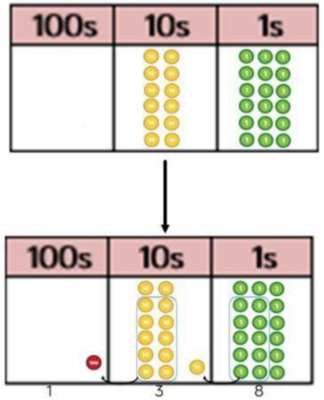
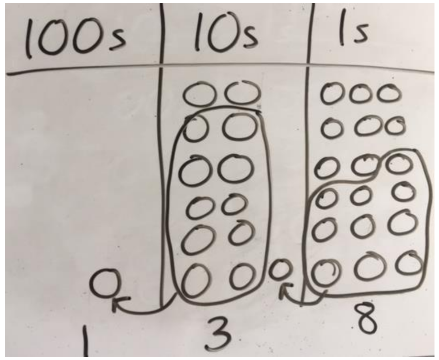
Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.  	Children to represent the practical resources in a picture and use a bar model.  	3 × 4 = 12 4 + 4 + 4 = 12

Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:  	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Partition to multiply using Numicon, base10or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3 × 23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 3 × 20 = 60 3 × 3 = 9 20 3 60 + 9 = 69 23 × 3 69

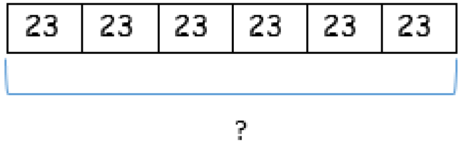
<table>
<tr>
<td>Formal column method with place value counters. 6 x 23

</td>
<td>Children to represent the counters/base 10, pictorially
e.g. the image below.

</td>
<td>Formal written method
6 x 23 =

  23
× 6
138
1 1</td>
</tr>
<tr>
<td colspan="2">When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.</td>
<td>   1 2 4
×   2 6
   7 4 4
    1 2
2 4 8 0
3 2 2 4
1 1

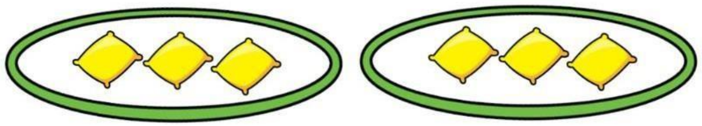
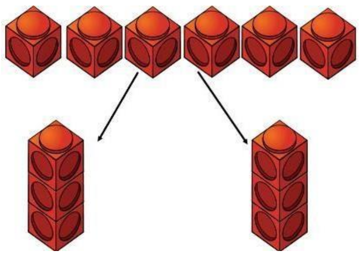
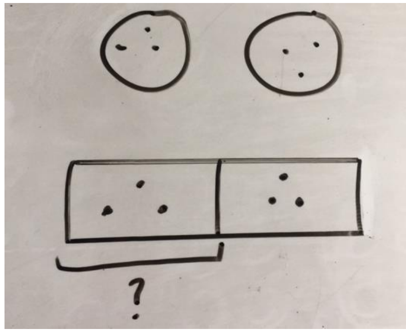
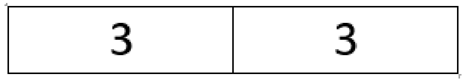
Answer: 3224</td>
</tr>
</table>

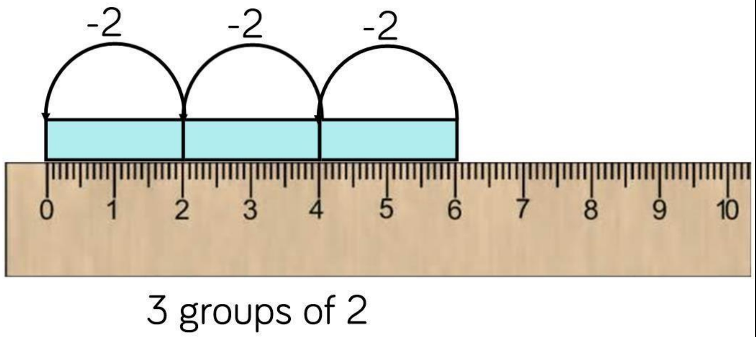
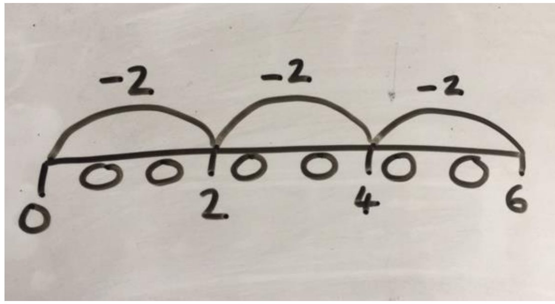
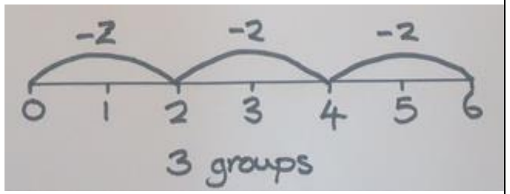
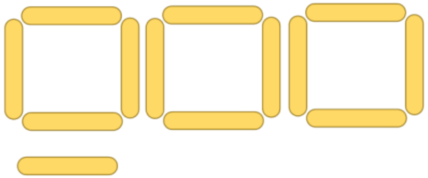
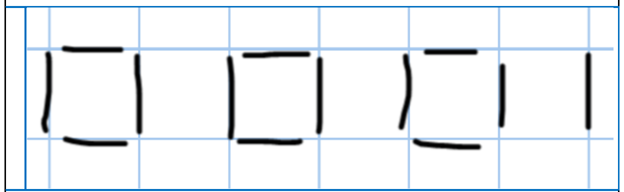
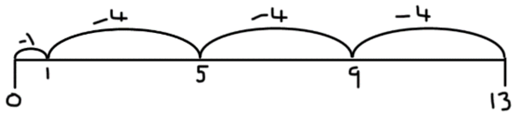
Conceptual variation; different ways to ask children to solve 6 ×23

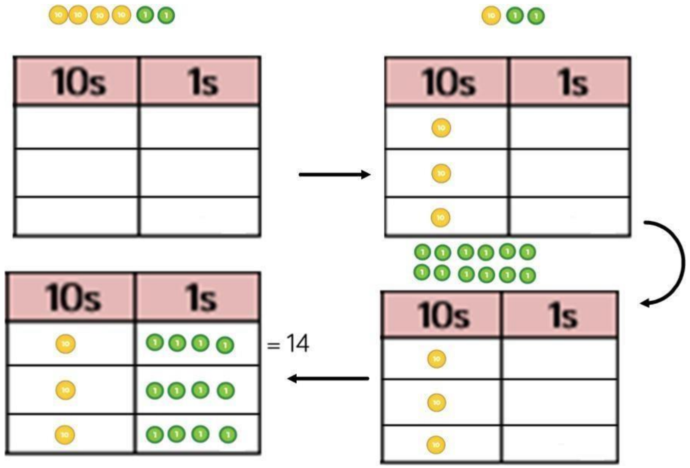
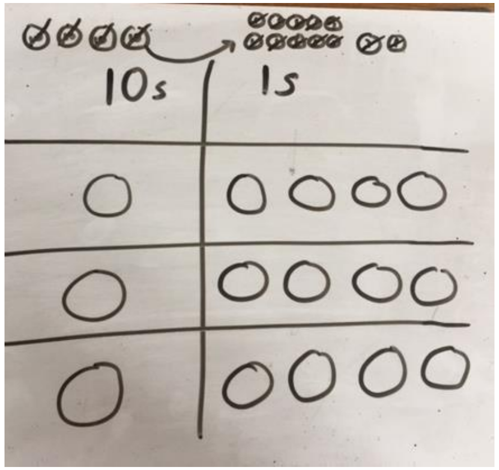
	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that 6 x 23 = 138	Findtheproductof6 and23 6 × 23= = 6 × 23 6 × 23 = ___ 23 × 6 = ___	What is the calculation? What is the product? 

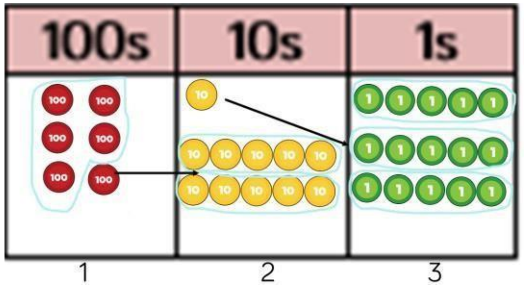
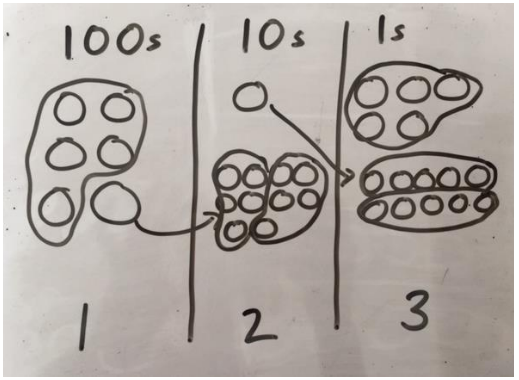
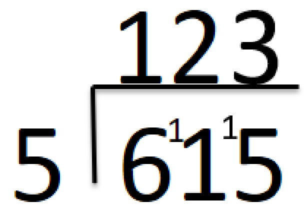
Calculation policy: Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2  	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.

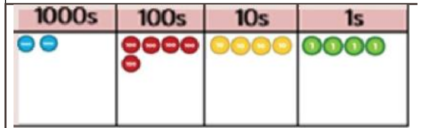
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  	Children to represent repeated subtraction pictorially.  	Abstract number line to represent the equal groups that have been subtracted. 
2d÷1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. ‘3 groups of 4, with 1 left over’ 

Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value Counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

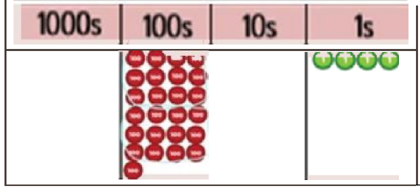
Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Children to the calculation using the short division scaffold. 

Long division using place value counters

2544 :-12|

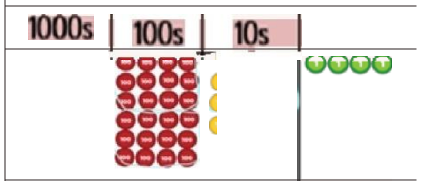


We can't group 2 thousands into groups of 12 so will exchange them.



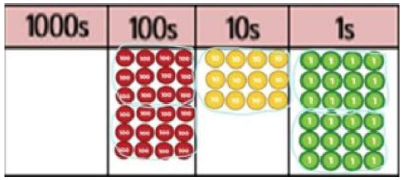
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

0 2
12|2544
24



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

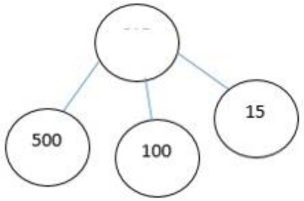
i2/ 2'544
2x
14



After exchanging the 2 tens, we have 24 ones. We can group 2A ones into 2 group of 12, which leaves no remainder

0 2 1 2
12 2544
24
14
12
24
24
0

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the pan whole model below, how can you divide 615 by 5 without using shon division?  	I have £615 and share it equally between 5 bank accounts How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	5 1615 615 ÷ 5 = i-e [ ] i = 615 :— 5	What is the calculation? What is the answer?